CHAPTER 2

POLYNOMIALS

(A) Main Concepts and Results

- Meaning of a Polynomial
- Degree of a polynomial
- Coefficients
- Monomials, Binomials etc.
- Constant, Linear, Quadratic Polynomials etc.
- Value of a polynomial for a given value of the variable
- Zeroes of a polynomial
- Remainder theorem
- Factor theorem
- Factorisation of a quadratic polynomial by splitting the middle term Factorisation of algebraic expressions by using the Factor theorem Algebraic identities –

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$$(x + y)^{2} = x^{2} + 2xy + y^{2}$$

$$(x - y)^{2} = x^{2} - 2xy + y^{2}$$

$$x^{2} - y^{2} = (x + y) (x - y)$$

$$(x + a) (x + b) = x^{2} + (a + b) x + ab$$

$$(x + y + z)^{2} = x^{2} + y^{2} + z^{2} + 2xy + 2yz + 2zx$$

$$(x + y)^{3} = x^{3} + 3x^{2}y + 3xy^{2} + y^{3} = x^{3} + y^{3} + 3xy (x + y)$$

$$(x - y)^{3} = x^{3} - 3x^{2}y + 3xy^{2} - y^{3} = x^{3} - y^{3} - 3xy (x - y)$$

(D)

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 $8\sqrt{2}+1$

$$x^{3} + y^{3} = (x + y) (x^{2} - xy + y^{2})$$

$$x^{3} - y^{3} = (x - y) (x^{2} + xy + y^{2})$$

$$x^{3} + y^{3} + z^{3} - 3xyz = (x + y + z) (x^{2} + y^{2} + z^{2} - xy - yz - zx)$$

(B) Multiple Choice Questions

 Sample Question 1 : If $x^2 + kx + 6 = (x + 2) (x + 3)$ for all x, then the value of k is

 (A) 1
 (B) -1
 (C) 5
 (D) 3

 Solution : Answer (C)

EXERCISE 2.1

Write the correct answer in each of the following :

1. Which one of the following is a polynomial?

(A)
$$\frac{x^2}{2} - \frac{2}{x^2}$$

(B) $\sqrt{2x} - 1$
(C) $x^2 + \frac{3x^2}{\sqrt{x}}$
(D) $\frac{x-1}{x+1}$

2. $\sqrt{2}$ is a polynomial of degree

(A) 2 (B) 0 (C) 1 (D)
$$\frac{1}{2}$$

3. Degree of the polynomial $4x^4 + 0x^3 + 0x^5 + 5x + 7$ is (A) 4 (B) 5 (C) 3

4. Degree of the zero polynomial is

(A) 0(B) 1(C) Any natural number(D) Not defined

5. If
$$p(x) = x^2 - 2\sqrt{2}x + 1$$
, then $p(2\sqrt{2})$ is equal to
(A) 0 (B) 1 (C) $4\sqrt{2}$

6. The value of the polynomial $5x - 4x^2 + 3$, when x = -1 is (A) -6 (B) 6 (C) 2 (D) -2



7.	7. If $p(x) = x + 3$, then $p(x) + p(-x)$ is equal to							
	(A)	3	(B)	2x	(C)	0	(D)	6
8. Zero of the zero polynomial is								
	(A)	0			(B)	1		
	(C)	Any real num	nber		(D)	Not defined		
9.	9. Zero of the polynomial $p(x) = 2x + 5$ is							
		2		5		2		5
	(A)	$-\frac{2}{5}$	(B)	$-\frac{3}{2}$	(C)	<u>-</u> 5	(D)	$\frac{3}{2}$
10. One of the zeroes of the polynomial $2x^2 + 7x - 4$ is								
	(A)	2	(B)	$\frac{1}{2}$	(C)	$-\frac{1}{2}$	(D)	-2
	F 2 51			2		2		
11. If $x^{31} + 51$ is divided by $x + 1$, the remainder is								
	(A)	0	(B)	1	(C)	49	(D)	50
12	If $x +$	- 1 is a factor o	of the p	olynomial 2.	$x^2 + kx$,	then the value	of <i>k</i> 18	_
	(A)	-3	(B)	4	(C)	2	(D)	-2
13. $x + 1$ is a factor of the polynomial								
	(A)	$x^3 + x^2 - x +$	1		(B)	$x^3 + x^2 + x + 1$		
	(C) $x^4 + x^3 + x^2 + 1$				(D)	$x^4 + 3x^3 + 3x^2 + x + 1$		
14. One of the factors of $(25x^2 - 1) + (1 + 5x)^2$ is								
	(A)	5 + x	(B)	5-x	(C)	5x - 1	(D)	10x
15. The value of $249^2 - 248^2$ is								
	(A)	1^{2}	(B)	477	(C)	487	(D)	497
16. The factorisation of $4x^2 + 8x + 3$ is								
	(A)	(x+1)(x+3)	3)		(B)	(2x + 1) (2x +	3)	
	(C)	(2x+2)(2x+2)	+ 5)		(D)	(2x - 1) (2x - 3))	
17. Which of the following is a factor of $(x + y)^3 - (x^3 + y^3)$?								
	(A)	$x^2 + y^2 + 2xy$	(B) <i>x</i>	$x^2 + y^2 - xy$	(C)	xy^2	(D)	3 <i>xy</i>
18. The coefficient of x in the expansion of $(x + 3)^3$ is								
	(A)	1	(B)	9	(C)	18	(D)	27
x y								
19. If $\frac{-1}{y} + \frac{-1}{x} = -1$ (x, y \neq 0), the value of $x^3 - y^3$ is								

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(A) 1 (B)
$$-1$$
 (C) 0 (D) $\frac{1}{2}$

20. If
$$49x^2 - b = \left(7x + \frac{1}{2}\right)\left(7x - \frac{1}{2}\right)$$
, then the value of *b* is

(A) 0 (B)
$$\frac{1}{\sqrt{2}}$$
 (C) $\frac{1}{4}$ (D) $\frac{1}{2}$

21. If
$$a + b + c = 0$$
, then $a^3 + b^3 + c^3$ is equal to
(A) 0 (B) abc (C) $3abc$ (D) $2abc$

(C) Short Answer Questions with Reasoning

(1

Sample Question 1 : Write whether the following statements are **True** or **False**. Justify your answer.

(i)
$$\frac{1}{\sqrt{5}}x^{\frac{1}{2}} + 1$$
 is a polynomial (ii) $\frac{6\sqrt{x} + x^{\frac{3}{2}}}{\sqrt{x}}$ is a polynomial, $x \neq 0$

Solution :

(i) False, because the exponent of the variable is not a whole number.

(ii) True, because
$$\frac{6\sqrt{x} + x^{\frac{3}{2}}}{\sqrt{x}} = 6 + x$$
, which is a polynomial.

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EXERCISE 2.2

1. Which of the following expressions are polynomials? Justify your answer:

(i) 8
(ii)
$$\sqrt{3}x^2 - 2x$$

(iii) $1 - \sqrt{5x}$
(iv) $\frac{1}{5x^{-2}} + 5x + 7$
(v) $\frac{(x-2)(x-4)}{x}$
(vi) $\frac{1}{x+1}$
(vii) $\frac{1}{7}a^3 - \frac{2}{\sqrt{3}}a^2 + 4a - 7$
(viii) $\frac{1}{2x}$

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- (i) A binomial can have atmost two terms
- (ii) Every polynomial is a binomial
- (iii) A binomial may have degree 5
- (iv) Zero of a polynomial is always 0
- (v) A polynomial cannot have more than one zero
- (vi) The degree of the sum of two polynomials each of degree 5 is always 5.

(D) Short Answer Questions

Sample Question 1 :

- (i) Check whether p(x) is a multiple of g(x) or not, where $p(x) = x^3 x + 1$, g(x) = 2 3x
- (ii) Check whether g(x) is a factor of p(x) or not, where

$$p(x) = 8x^3 - 6x^2 - 4x + 3, \quad g(x) = \frac{x}{3} - \frac{1}{4}$$

Solution :

(i) p(x) will be a multiple of g(x) if g(x) divides p(x).

Now, g(x) = 2 - 3x = 0 gives $x = \frac{2}{3}$

Remainder

$$= p\left(\frac{2}{3}\right) = \left(\frac{2}{3}\right)^3 - \left(\frac{2}{3}\right) + 1$$

$$=\frac{8}{27}-\frac{2}{3}+1=\frac{17}{27}$$

Since remainder $\neq 0$, so, p(x) is not a multiple of g(x).

(ii)
$$g(x) = \frac{x}{3} - \frac{1}{4} = 0$$
 gives $x = \frac{3}{4}$

g(x) will be a factor of p(x) if $p\left(\frac{3}{4}\right) = 0$ (Factor theorem)

Now, $p\left(\frac{3}{4}\right) = 8\left(\frac{3}{4}\right)^3 - 6\left(\frac{3}{4}\right)^2 - 4\left(\frac{3}{4}\right) + 3$



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EXEMPLAR PROBLEMS

$$= 8 \times \frac{27}{64} - 6 \times \frac{9}{16} - 3 + 3 = 0$$

Since, $p\left(\frac{3}{4}\right) = 0$, so, g(x) is a factor of p(x).

Sample Question 2 : Find the value of *a*, if x - a is a factor of $x^3 - ax^2 + 2x + a - 1$. Solution : Let $p(x) = x^3 - ax^2 + 2x + a - 1$ Since x - a is a factor of p(x), so p(a) = 0. i.e., $a^3 - a(a)^2 + 2a + a - 1 = 0$ $a^3 - a^3 + 2a + a - 1 = 0$ 3a = 1

Therefore, $a = \frac{1}{3}$

Sample Question 3 : (i) Without actually calculating the cubes, find the value of $48^3 - 30^3 - 18^3$.

(ii)Without finding the cubes, factorise $(x - y)^3 + (y - z)^3 + (z - x)^3$. **Solution :** We know that $x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - zx)$. If x + y + z = 0, then $x^3 + y^3 + z^3 - 3xyz = 0$ or $x^3 + y^3 + z^3 = 3xyz$.

- (i) We have to find the value of 48³ 30³ 18³ = 48³ + (-30)³ + (-18)³. Here, 48 + (-30) + (-18) = 0 So, 48³ + (-30)³ + (-18)³ = 3 × 48 × (-30) × (-18) = 77760
- (ii) Here, (x y) + (y z) + (z x) = 0Therefore, $(x - y)^3 + (y - z)^3 + (z - x)^3 = 3(x - y)(y - z)(z - x).$

EXERCISE 2.3

- 1. Classify the following polynomials as polynomials in one variable, two variables etc.
 - (i) $x^2 + x + 1$ (ii) $y^3 5y$
 - (iii) xy + yz + zx (iv) $x^2 2xy + y^2 + 1$

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- 2. Determine the degree of each of the following polynomials :
 - (i) 2x-1 (ii) -10
 - (iii) $x^3 9x + 3x^5$ (iv) $y^3 (1 y^4)$
- 3. For the polynomial

$$\frac{x^3 + 2x + 1}{5} - \frac{7}{2}x^2 - x^6$$
, write

- (i) the degree of the polynomial
- (ii) the coefficient of x^3
- (iii) the coefficient of x^6
- (iv) the constant term
- 4. Write the coefficient of x^2 in each of the following :
 - (i) $\frac{\pi}{6}x + x^2 1$ (ii) 3x 5

(iii)
$$(x-1)(3x-4)$$
 (iv) $(2x-5)(2x^2-3x+1)$

5. Classify the following as a constant, linear, quadratic and cubic polynomials :

- (i) $2 x^2 + x^3$ (ii) $3x^3$ (iii) $5t \sqrt{7}$ (iv) $4 5y^2$ (v) 3 (vi) 2 + x (vii) $y^3 - y$ (viii) $1 + x + x^2$ (ix) t^2 (x) $\sqrt{2x-1}$
- 6. Give an example of a polynomial, which is :
 - (i) monomial of degree 1
 - (ii) binomial of degree 20
 - (iii) trinomial of degree 2
- 7. Find the value of the polynomial $3x^3 4x^2 + 7x 5$, when x = 3 and also when x = -3.
- 8. If $p(x) = x^2 4x + 3$, evaluate : $p(2) p(-1) + p\left(\frac{1}{2}\right)$
- **9.** Find p(0), p(1), p(-2) for the following polynomials :

$$p(x) = 10x - 4x^2 - 3$$
 (ii) $p(y) = (y + 2)(y - 2)$

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10. Verify whether the following are True or False :

(i) -3 is a zero of x - 3

(i)

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(ii)
$$-\frac{1}{3}$$
 is a zero of $3x + 1$

(iii)
$$\frac{-4}{5}$$
 is a zero of $4-5y$

- (iv) 0 and 2 are the zeroes of $t^2 2t$
- (v) -3 is a zero of $y^2 + y 6$
- **11.** Find the zeroes of the polynomial in each of the following :
 - (i) p(x) = x 4(ii) g(x) = 3 - 6x
 - (iii) q(x) = 2x 7(iv) h(y) = 2y
- **12.** Find the zeroes of the polynomial :

$$p(x) = (x - 2)^2 - (x + 2)^2$$

13. By actual division, find the quotient and the remainder when the first polynomial is divided by the second polynomial : $x^4 + 1$; x - 1

14. By Remainder Theorem find the remainder, when p(x) is divided by g(x), where

(i)
$$p(x) = x^3 - 2x^2 - 4x - 1$$
, $g(x) = x + 1$

(ii)
$$p(x) = x^3 - 3x^2 + 4x + 50$$
, $g(x) = x - 3$

(ii) $p(x) = x^3 - 3x^2 + 4x + 50$, g(x) = x - 3(iii) $p(x) = 4x^3 - 12x^2 + 14x - 3$, g(x) = 2x

(iv)
$$p(x) = x^3 - 6x^2 + 2x - 4$$
, $g(x) = 1 - \frac{3}{2}x$

15. Check whether p(x) is a multiple of g(x) or not :

- (i) $p(x) = x^3 5x^2 + 4x 3$, g(x) = x 2
- (ii) $p(x) = 2x^3 11x^2 4x + 5$, g(x) = 2x + 1

16. Show that :

- (i) x + 3 is a factor of $69 + 11x x^2 + x^3$.
- (ii) 2x 3 is a factor of $x + 2x^3 9x^2 + 12$.

17. Determine which of the following polynomials has x - 2 a factor :

(i)
$$3x^2 + 6x - 24$$
 (ii) $4x^2 + x - 2$

- **18.** Show that p 1 is a factor of $p^{10} 1$ and also of $p^{11} 1$.
- **19.** For what value of *m* is $x^3 2mx^2 + 16$ divisible by x + 2?
- **20.** If x + 2a is a factor of $x^5 4a^2x^3 + 2x + 2a + 3$, find a.
- **21.** Find the value of m so that 2x 1 be a factor of $8x^4 + 4x^3 16x^2 + 10x + m$.

22. If x + 1 is a factor of $ax^3 + x^2 - 2x + 4a - 9$, find the value of a. 23. Factorise : (i) $x^2 + 9x + 18$ $6x^2 + 7x - 3$ (ii) $2x^2 - 7x - 15$ $84 - 2r - 2r^2$ (iii) (iv) 24. Factorise : (ii) $x^3 - 6x^2 + 11x - 6$ (i) $2x^3 - 3x^2 - 17x + 30$ (iii) $x^3 + x^2 - 4x - 4$ $3x^3 - x^2 - 3x + 1$ (iv) **25.** Using suitable identity, evaluate the following: (i) 103^{3} (ii) 101×102 (iii) **999**² **26.** Factorise the following: (i) $4x^2 + 20x + 25$ (ii) $9y^2 - 66yz + 121z^2$ $\left(2x+\frac{1}{3}\right)^2 - \left(x-\frac{1}{2}\right)^2$ (iii) **27.** Factorise the following : (i) $9x^2 - 12x + 3$ (ii) -12x + 4**28.** Expand the following : (i) $(4a - b + 2c)^2$ (ii) $(3a - 5b - c)^2$ (iii) $(-x + 2y - 3z)^2$ **29.** Factorise the following : (i) $9x^2 + 4y^2 + 16z^2 + 12xy - 16yz - 24xz$ $25x^2 + 16y^2 + 4z^2 - 40xy + 16yz - 20xz$ (ii) $16x^2 + 4y^2 + 9z^2 - 16xy - 12yz + 24xz$ (iii) **30.** If a + b + c = 9 and ab + bc + ca = 26, find $a^2 + b^2 + c^2$. **31.** Expand the following : (ii) $\left(\frac{1}{x} + \frac{y}{3}\right)^3$ (iii) $\left(4 - \frac{1}{3x}\right)^3$ $(3a-2b)^3$ (i) **32.** Factorise the following : $1 - 64a^3 - 12a + 48a^2$ (i)

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(ii)
$$8p^3 + \frac{12}{5}p^2 + \frac{6}{25}p + \frac{1}{125}$$

33. Find the following products :

(i)
$$\left(\frac{x}{2}+2y\right)\left(\frac{x^2}{4}-xy+4y^2\right)$$
 (ii) $(x^2-1)(x^4+x^2+1)$

34. Factorise :

(i)
$$1 + 64x^3$$
 (ii) $a^3 - 2\sqrt{2}b^3$

35. Find the following product :

$$(2x - y + 3z) (4x2 + y2 + 9z2 + 2xy + 3yz - 6xz)$$

36. Factorise :

(i)
$$a^3 - 8b^3 - 64c^3 - 24abc$$
 (ii) $2\sqrt{2}a^3 + 8b^3 - 27c^3 + 18\sqrt{2}abc$

37. Without actually calculating the cubes, find the value of :

(i)
$$\left(\frac{1}{2}\right)^3 + \left(\frac{1}{3}\right)^3 - \left(\frac{5}{6}\right)^3$$

(ii)
$$(0.2)^3 - (0.3)^3 + (0.1)^3$$

38. Without finding the cubes, factorise

$$(x - 2y)^3 + (2y - 3z)^3 + (3z - x)^3$$

39. Find the value of

- (i) $x^3 + y^3 12xy + 64$, when x + y = -4
- (ii) $x^3 8y^3 36xy 216$, when x = 2y + 6
- **40.** Give possible expressions for the length and breadth of the rectangle whose area is given by $4a^2 + 4a 3$.

(E) Long Answer Questions

Sample Question 1 : If x + y = 12 and xy = 27, find the value of $x^3 + y^3$. Solution :

$$x^{3} + y^{3} = (x + y) (x^{2} - xy + y^{2})$$

= $(x + y) [(x + y)^{2} - 3xy]$
= $12 [12^{2} - 3 \times 27]$
= $12 \times 63 = 756$

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Alternative Solution :

 x^3

$$+ y^{3} = (x + y)^{3} - 3xy (x + y)$$

$$= 12^{3} - 3 \times 27 \times 12$$

$$= 12 [12^{2} - 3 \times 27]$$

$$= 12 \times 63 = 756$$

EXERCISE 2.4

- 1. If the polynomials $az^3 + 4z^2 + 3z 4$ and $z^3 4z + a$ leave the same remainder when divided by z 3, find the value of a.
- 2. The polynomial $p(x) = x^4 2x^3 + 3x^2 ax + 3a 7$ when divided by x + 1 leaves the remainder 19. Find the values of *a*. Also find the remainder when p(x) is divided by x + 2.
- 3. If both x 2 and $x \frac{1}{2}$ are factors of $px^2 + 5x + r$, show that p = r.
- 4. Without actual division, prove that $2x^4 5x^3 + 2x^2 x + 2$ is divisible by $x^2 3x + 2$. [Hint: Factorise $x^2 - 3x + 2$]
- 5. Simplify $(2x 5y)^3 (2x + 5y)^3$.
- 6. Multiply $x^2 + 4y^2 + z^2 + 2xy + xz 2yz$ by (-z + x 2y).
- 7. If a, b, c are all non-zero and a + b + c = 0, prove that $\frac{a^2}{bc} + \frac{b^2}{ca} + \frac{c^2}{ab} = 3$.
- 8. If a + b + c = 5 and ab + bc + ca = 10, then prove that $a^3 + b^3 + c^3 3abc = -25$.
- 9. Prove that $(a + b + c)^3 a^3 b^3 c^3 = 3(a + b) (b + c) (c + a)$.



